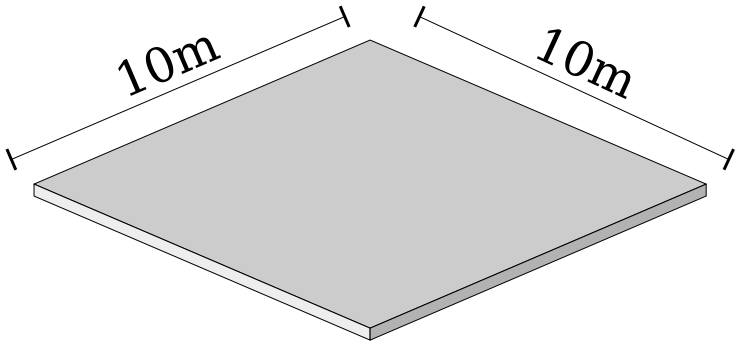
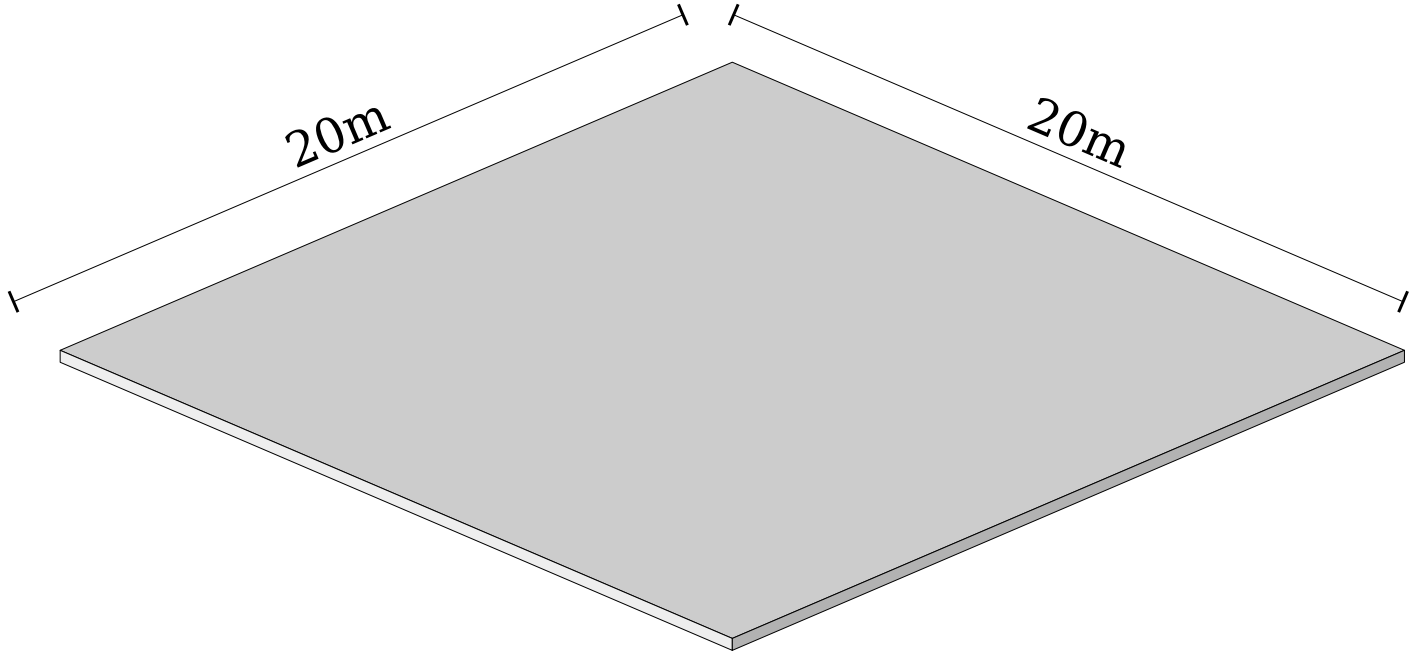
# Big-O Notation

# Estimating Quantities



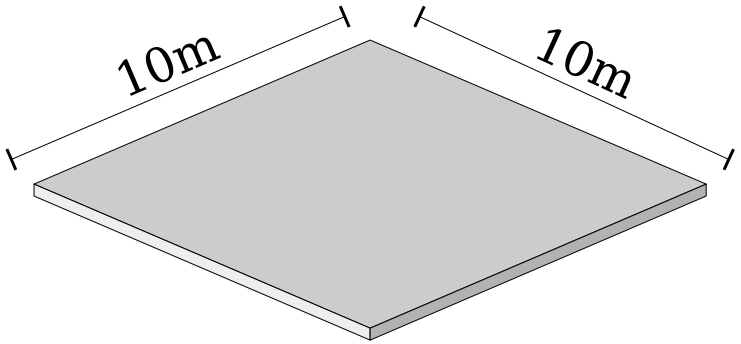

Mass: 100kg




---

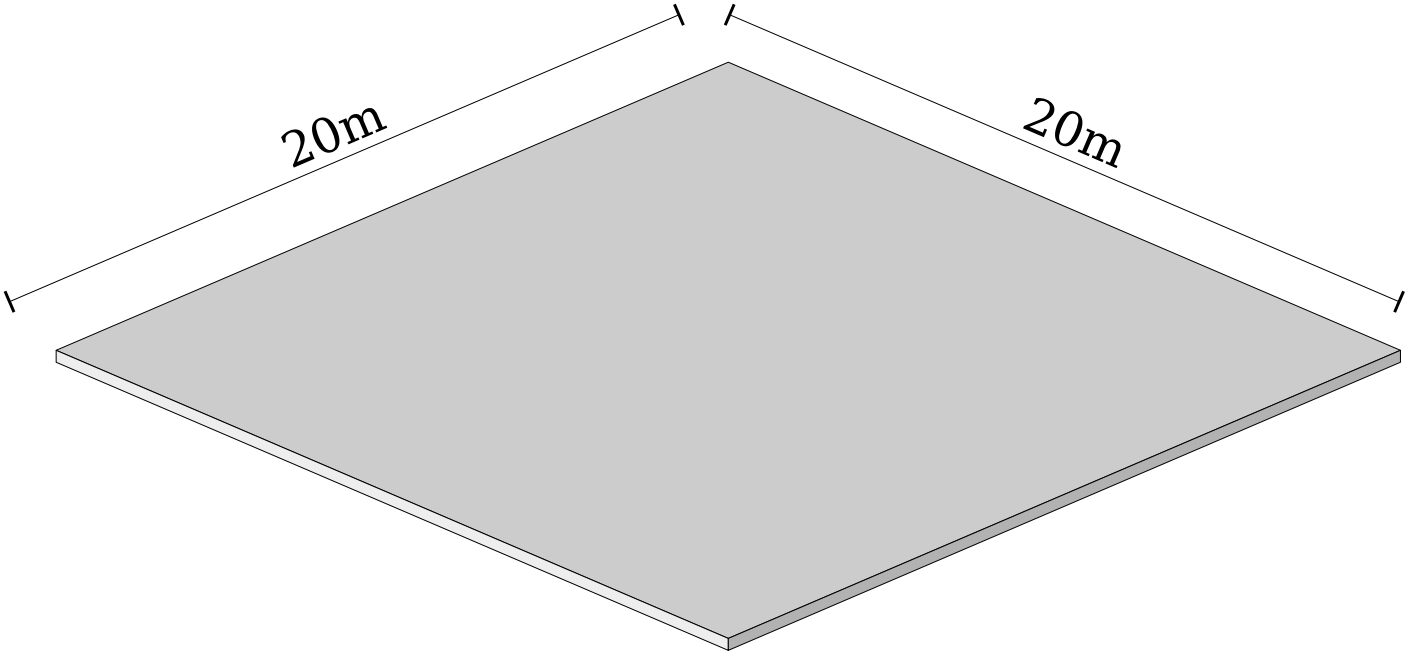
These two square plates are made of the same material.
They have the same thickness.

What’s your best guess for the mass of the second square?




Mass: 100kg

Answer at
***https://pollev.com/cs106bwin23***

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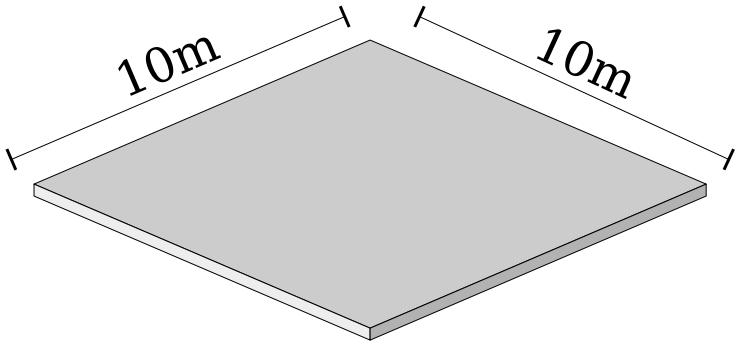
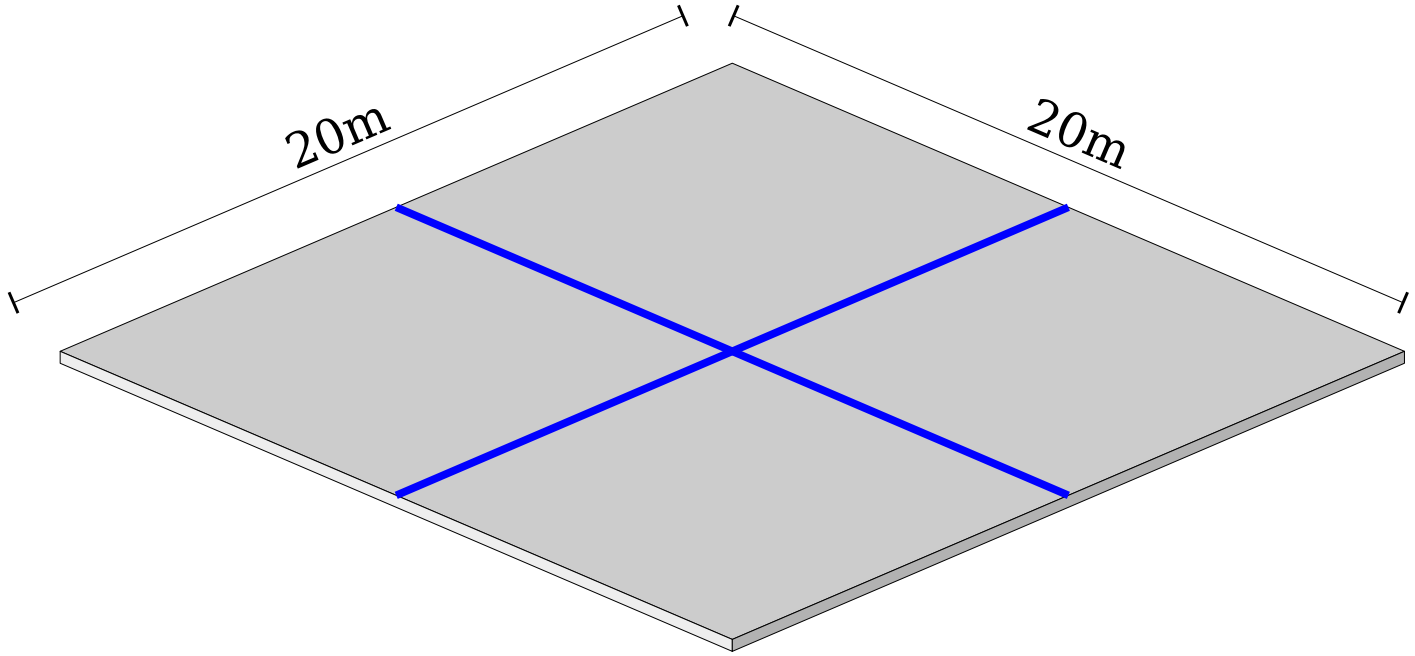



Mass: 100kg

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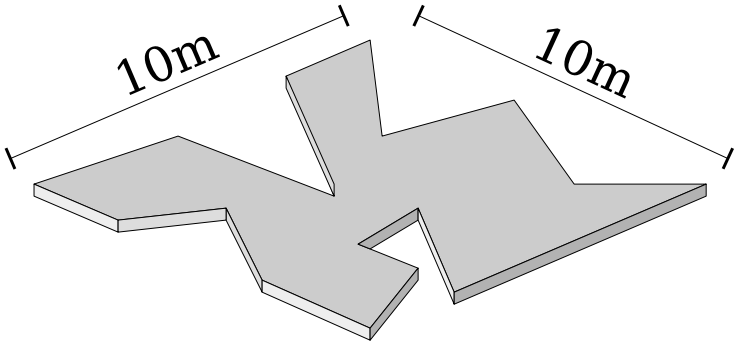
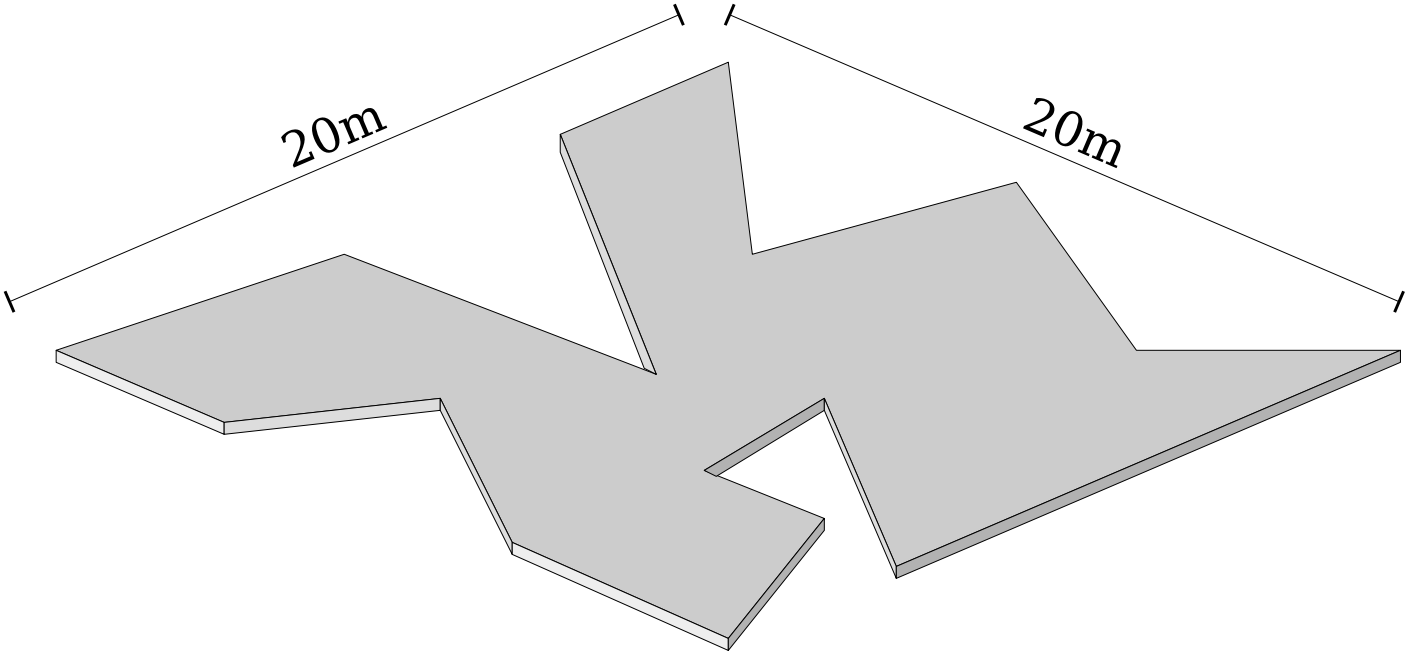



Mass: 60kg

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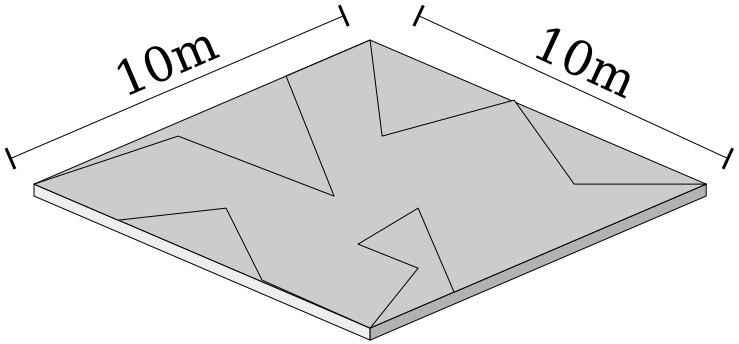
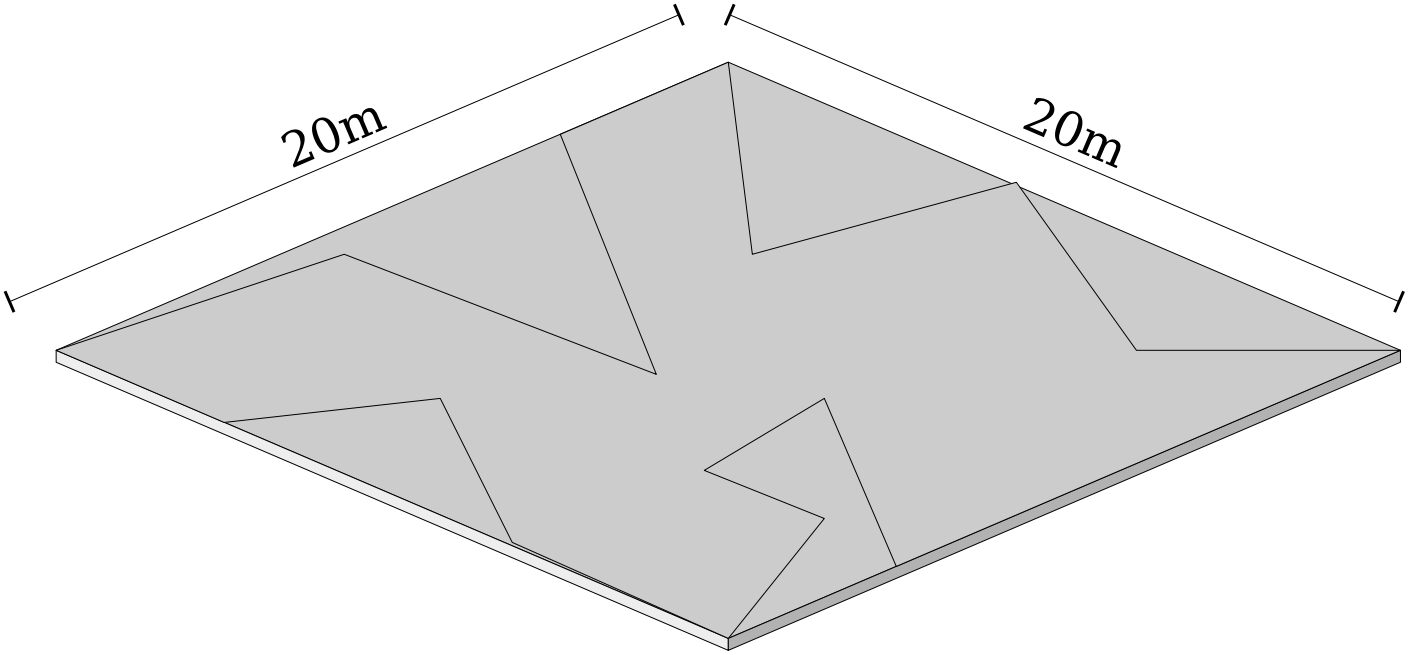
These two figures are made of the same material.
They have the same thickness.

What’s your best guess for the mass of the second figure?




Mass: 60kg

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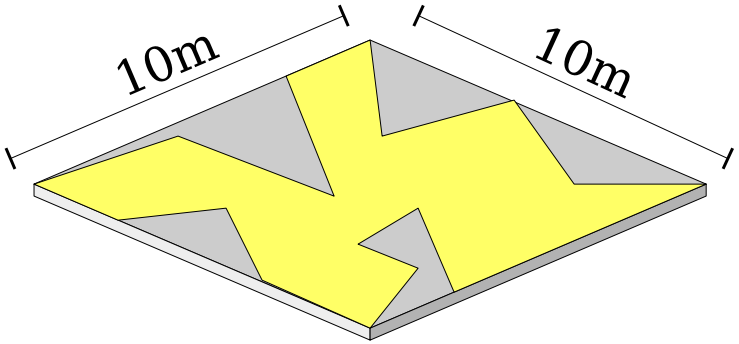


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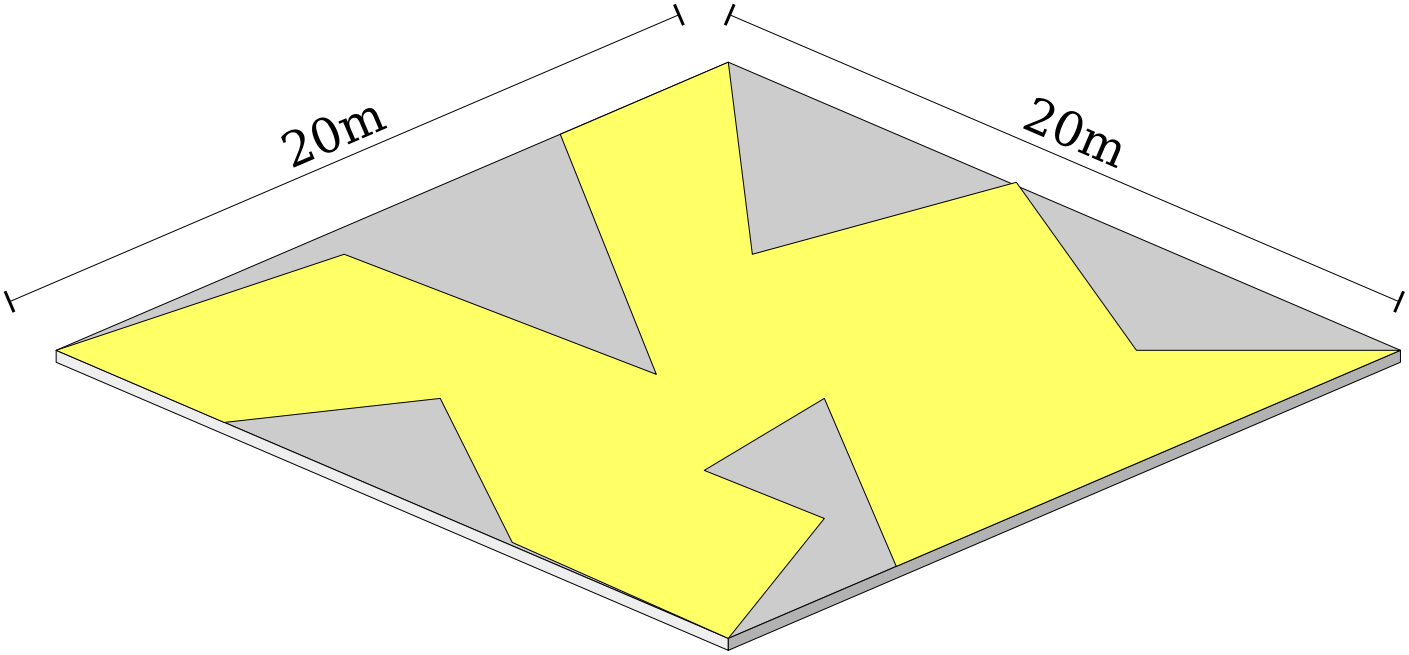


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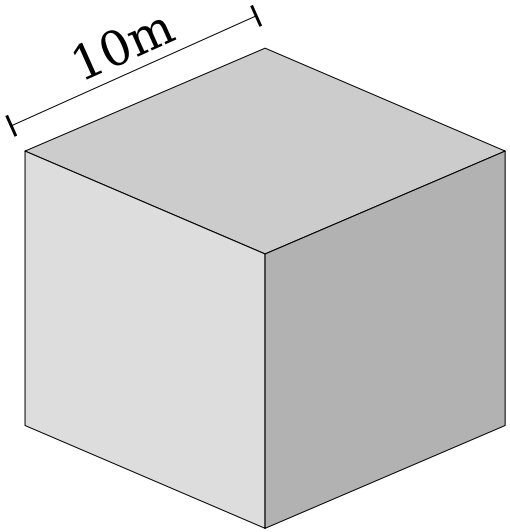
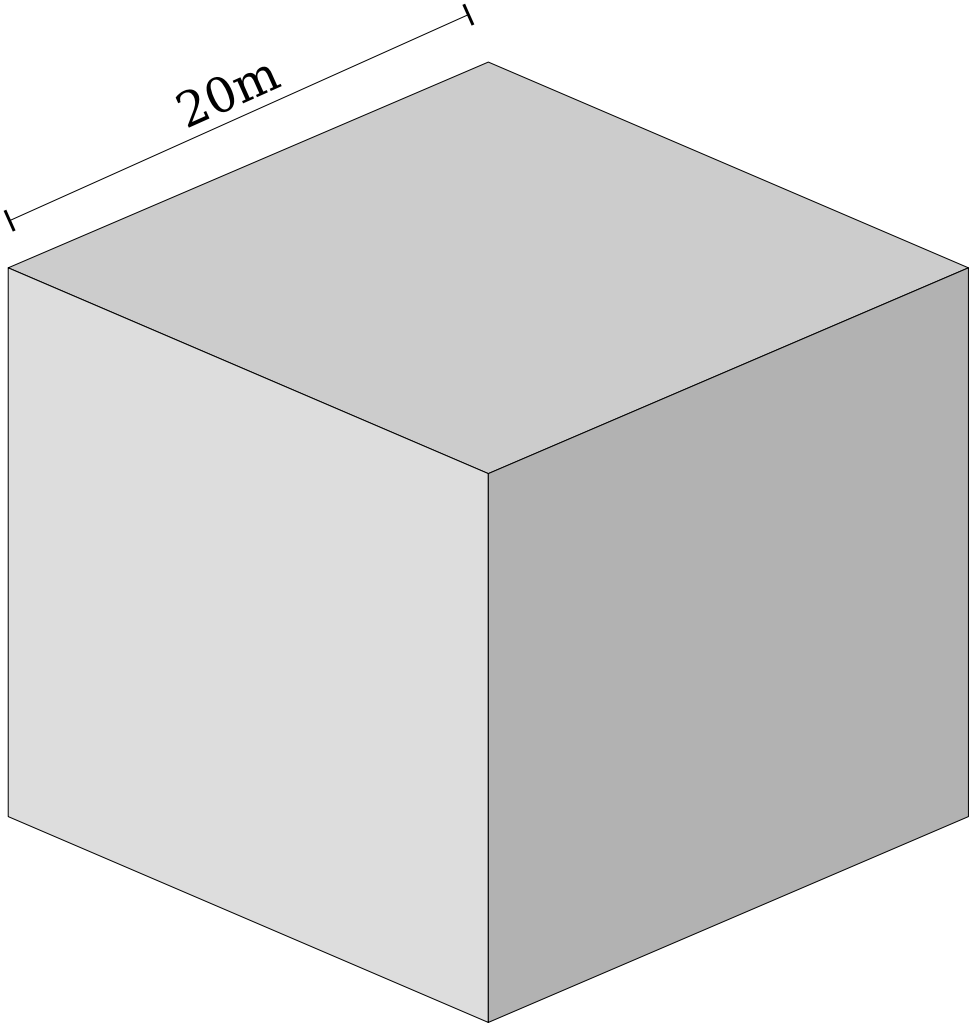



Mass: 100kg

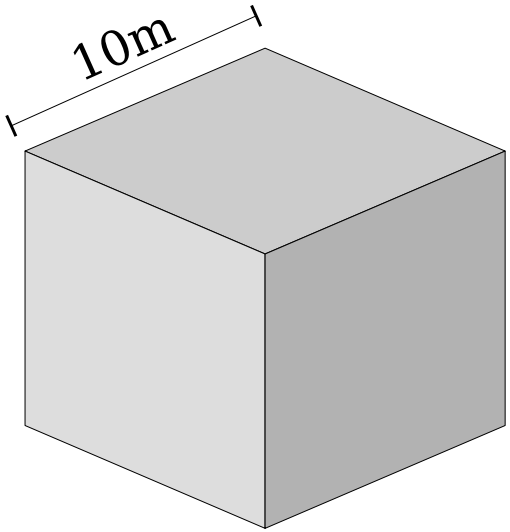



---

These two cubes are made of the same material.

What’s your best guess for the mass of the second cube?

Answer at
***https://pollev.com/cs106bwin23***

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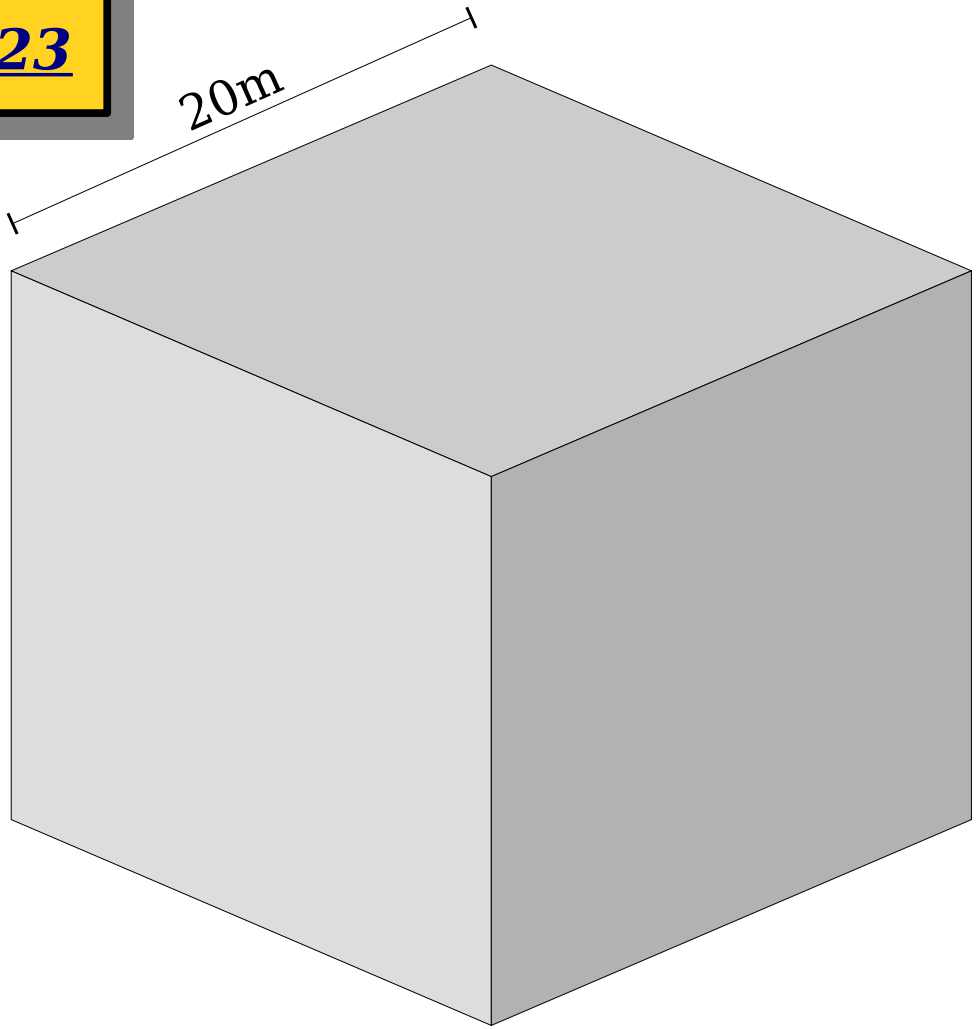


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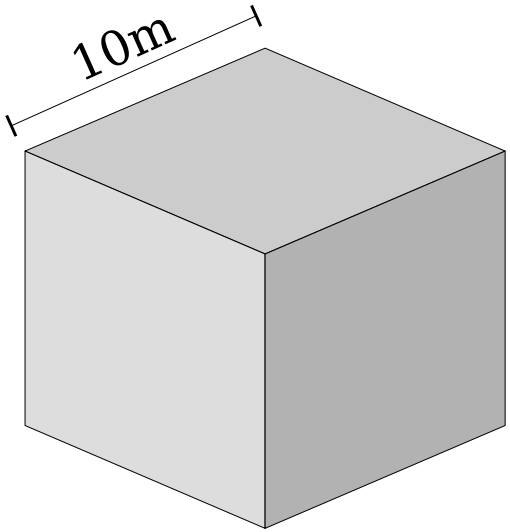


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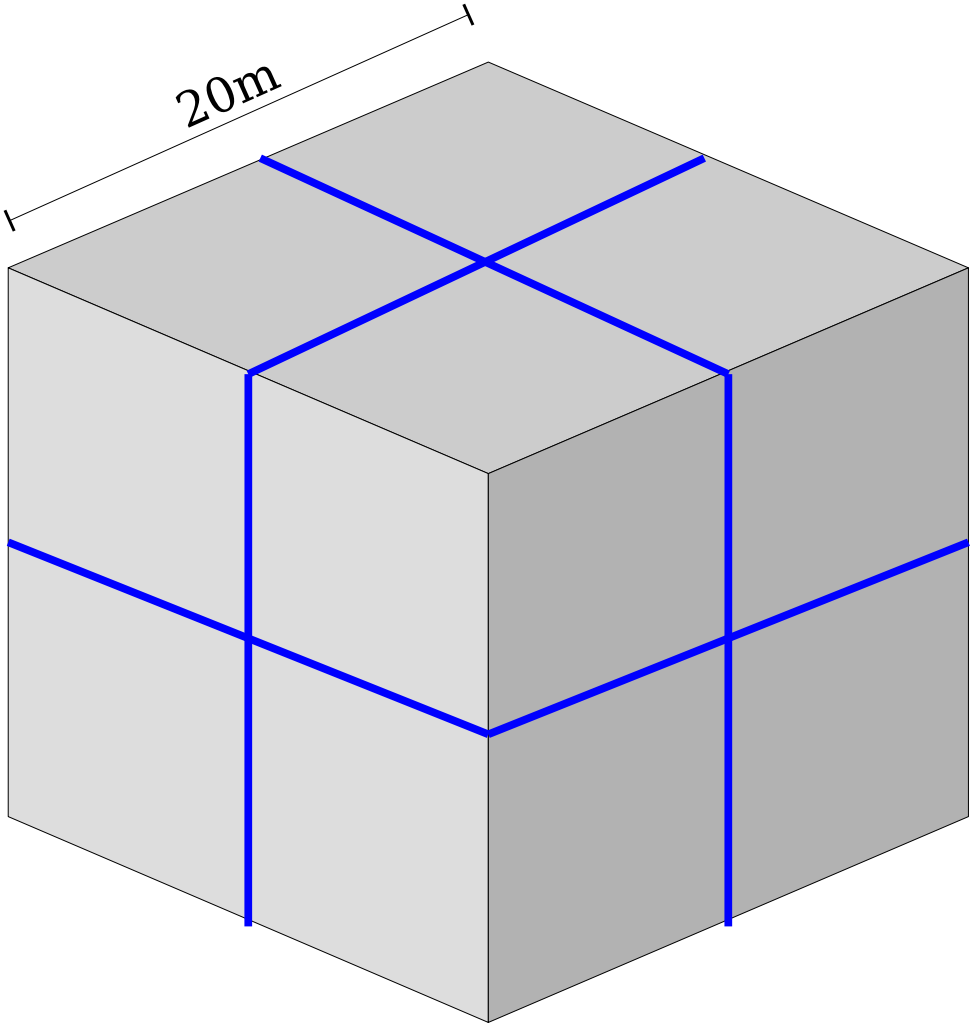


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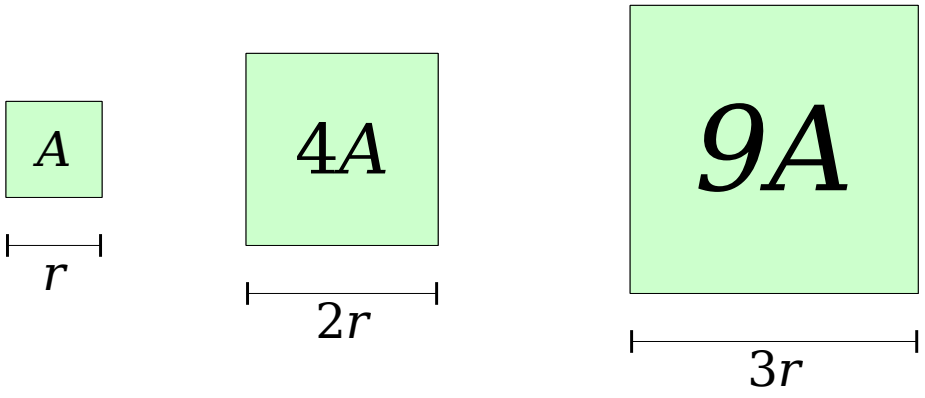
These two cubes are made of the same material.

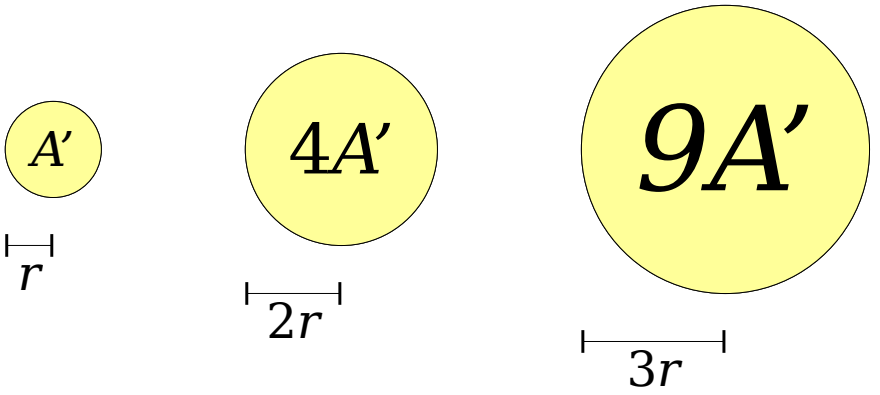
What’s your best guess for the mass of the second cube?

Knowing the rate at which some quantity scales allows you to predict its value in the future, even if you don’t have an exact formula.

# Big-O Notation

- ***Big-O notation*** is a way of quantifying the rate at which some quantity grows.
- For example:
  - A square of side length $r$ has area $O(r^2)$.
  - A circle of radius $r$ has area $O(r^2)$.




Doubling $r$ increases area 4×.
Tripling $r$ increases area 9×.

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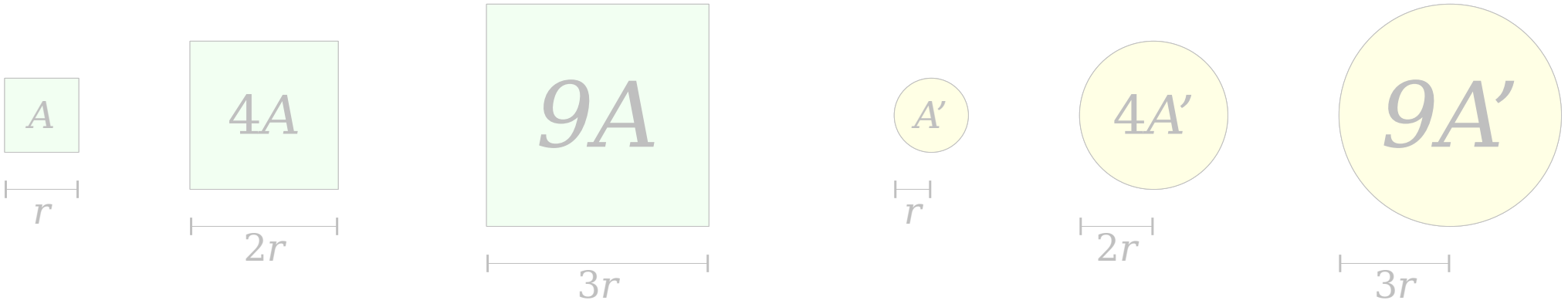


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This just says that these quantities grow at the same relative rates. It does not say that they're equal!

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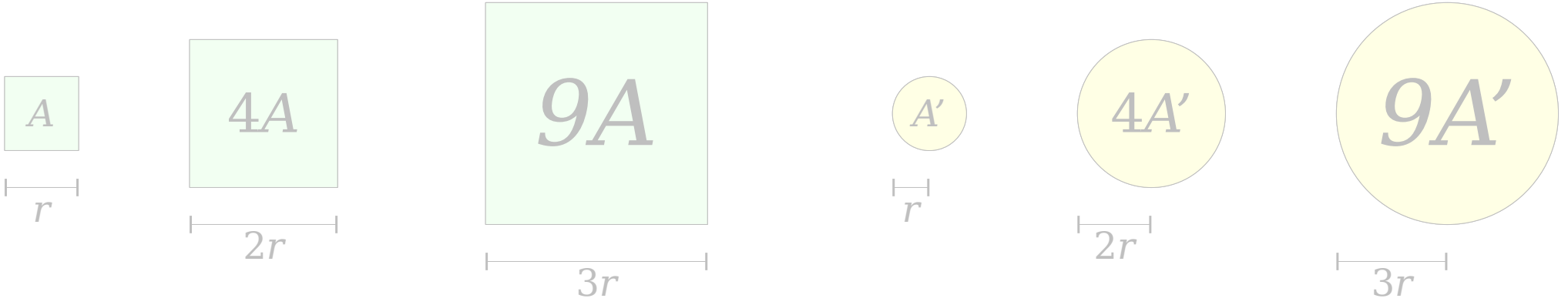


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  - A square of side length $r$ has area $O(r^2)$.
  - A circle of radius $r$ has area $O(r^2)$.
  - A cube of side length $r$ has volume $O(r^3)$.
  - A sphere of radius $r$ has volume $O(r^3)$.
  - A sphere of radius $r$ has surface area $O(r^2)$.
  - A cube of side length $r$ has surface area $O(r^2)$.

# Example: Network Value

- ***Metcalfe's Law*** says that

  The value of a communications network with $n$ users is $O(n^2)$.

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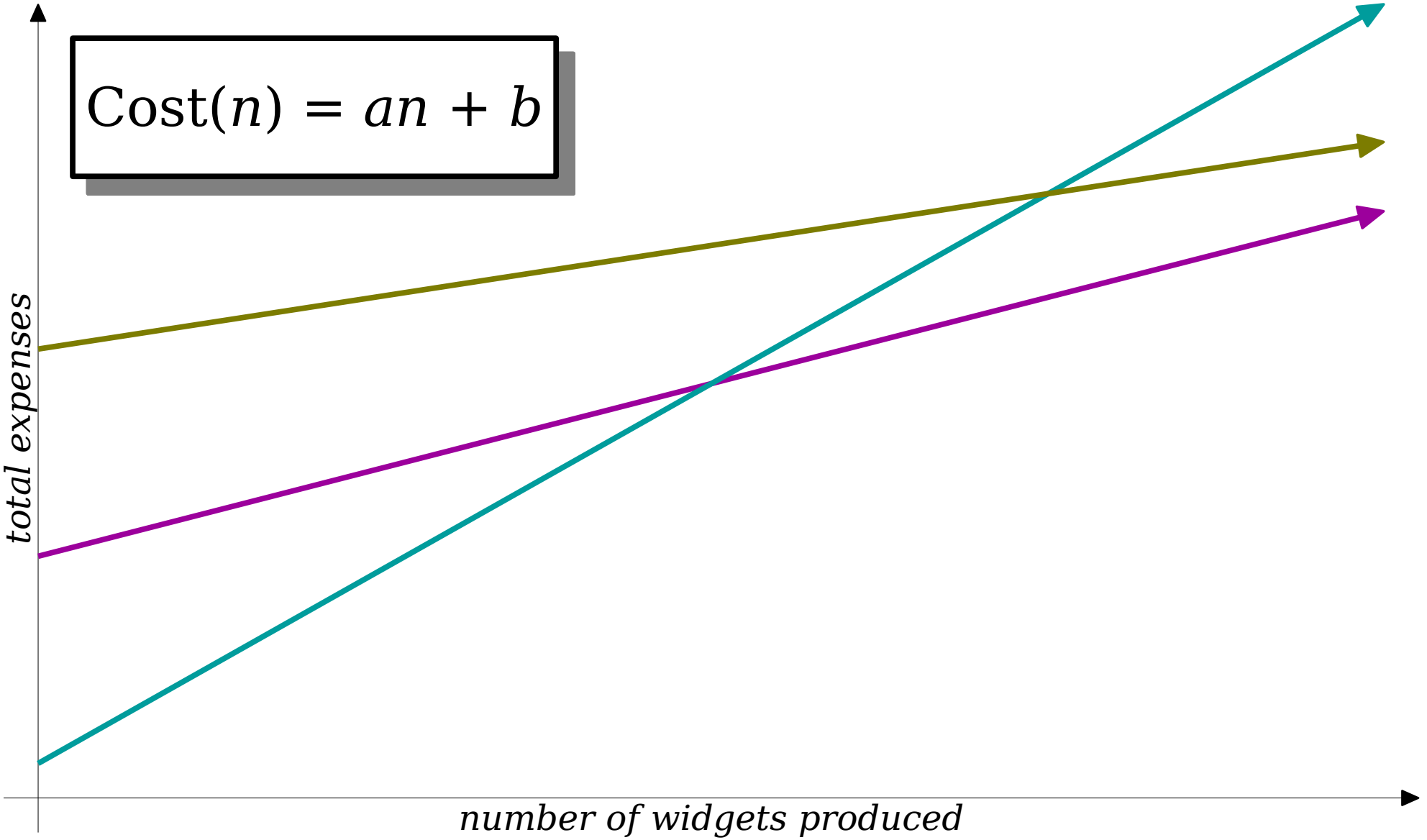
- Imagine a social network has 10,000,000 users and is worth $10,000,000. Estimate how many users it needs to have to be worth $1,000,000,000.

- ***Reasonable guess:*** The network needs to grow its value 100×. Since value grows quadratically with size, it needs to grow its user base 10×, requiring 100,000,000 users.

# A Messier Example: Manufacturing

# Making Widgets



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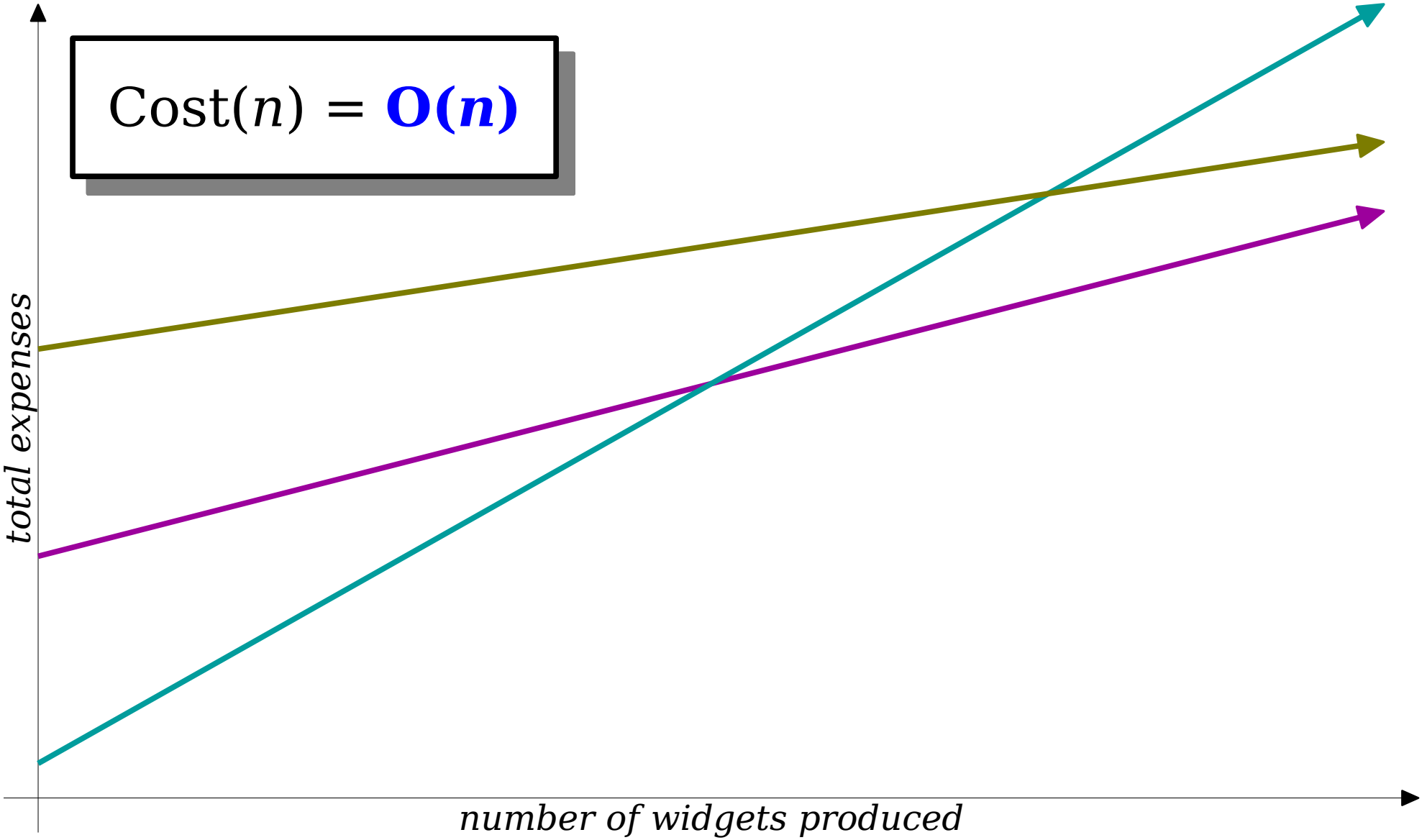

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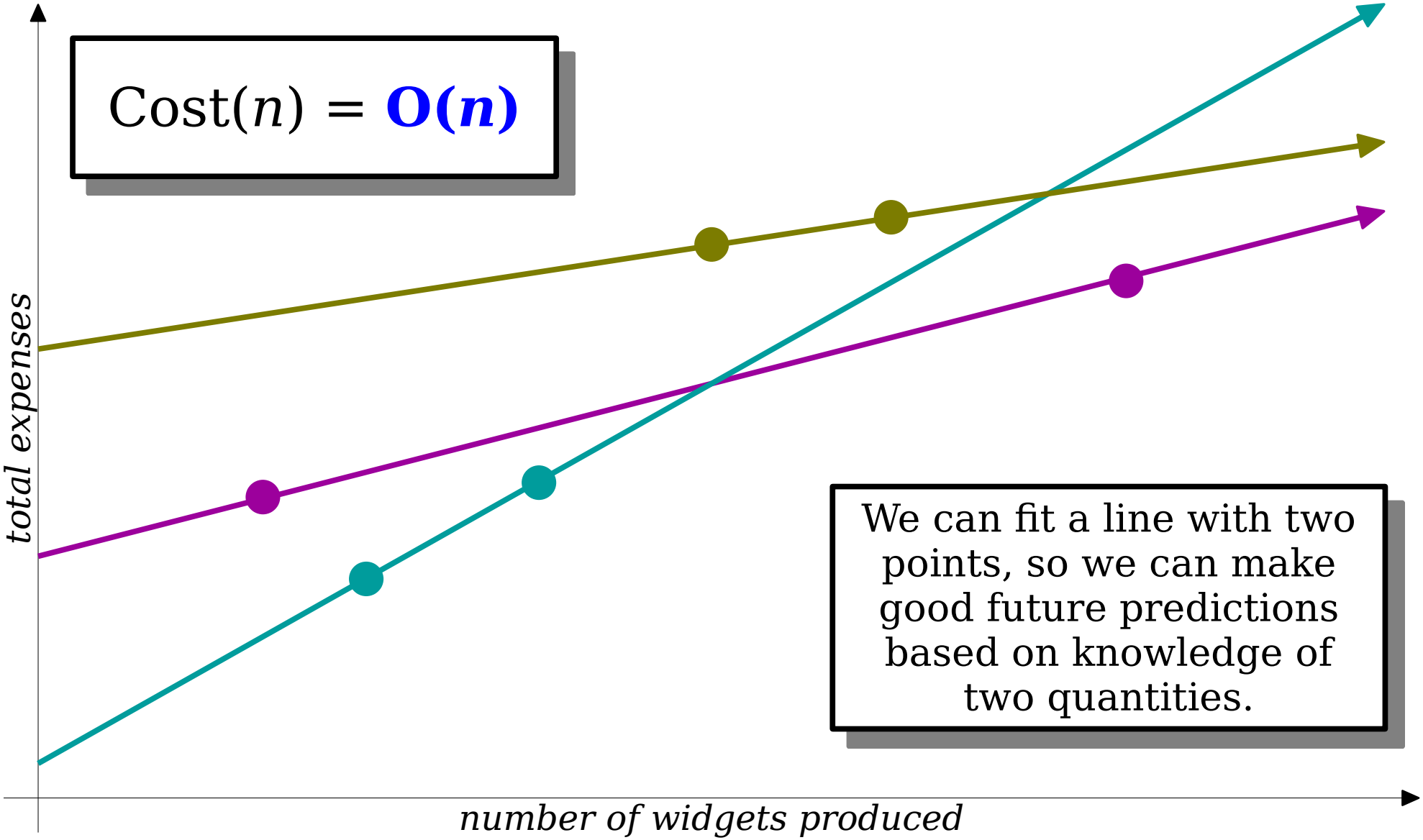

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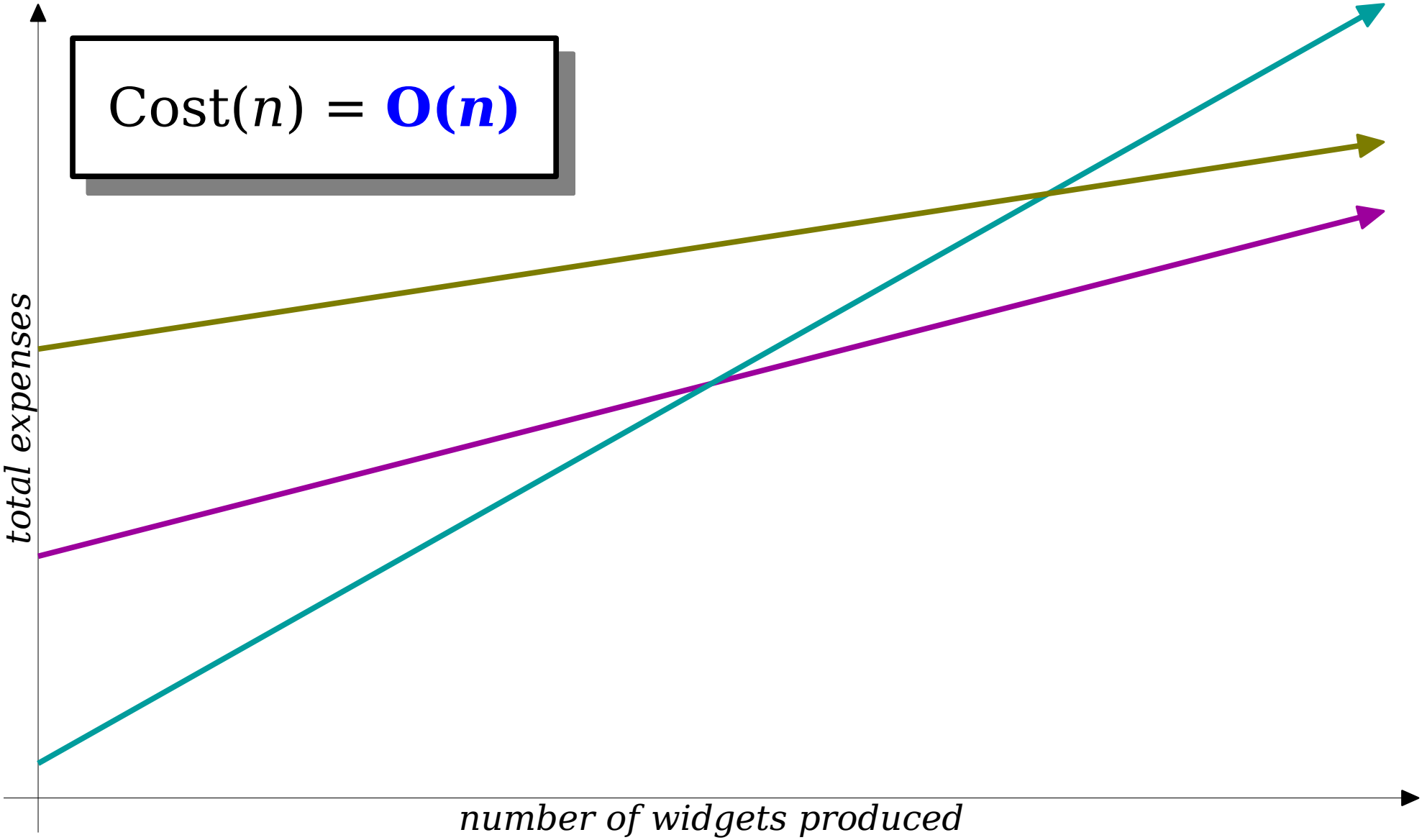

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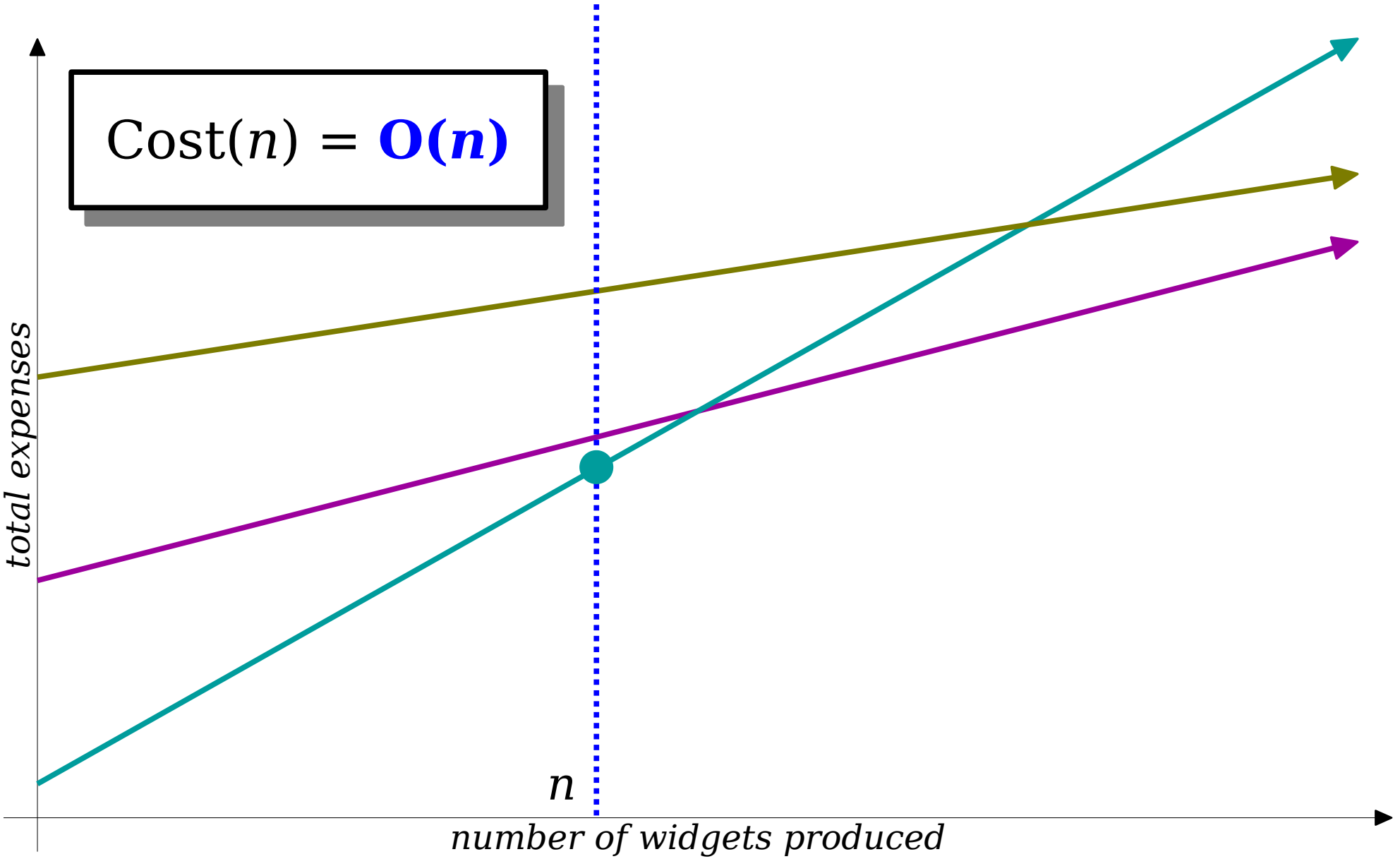

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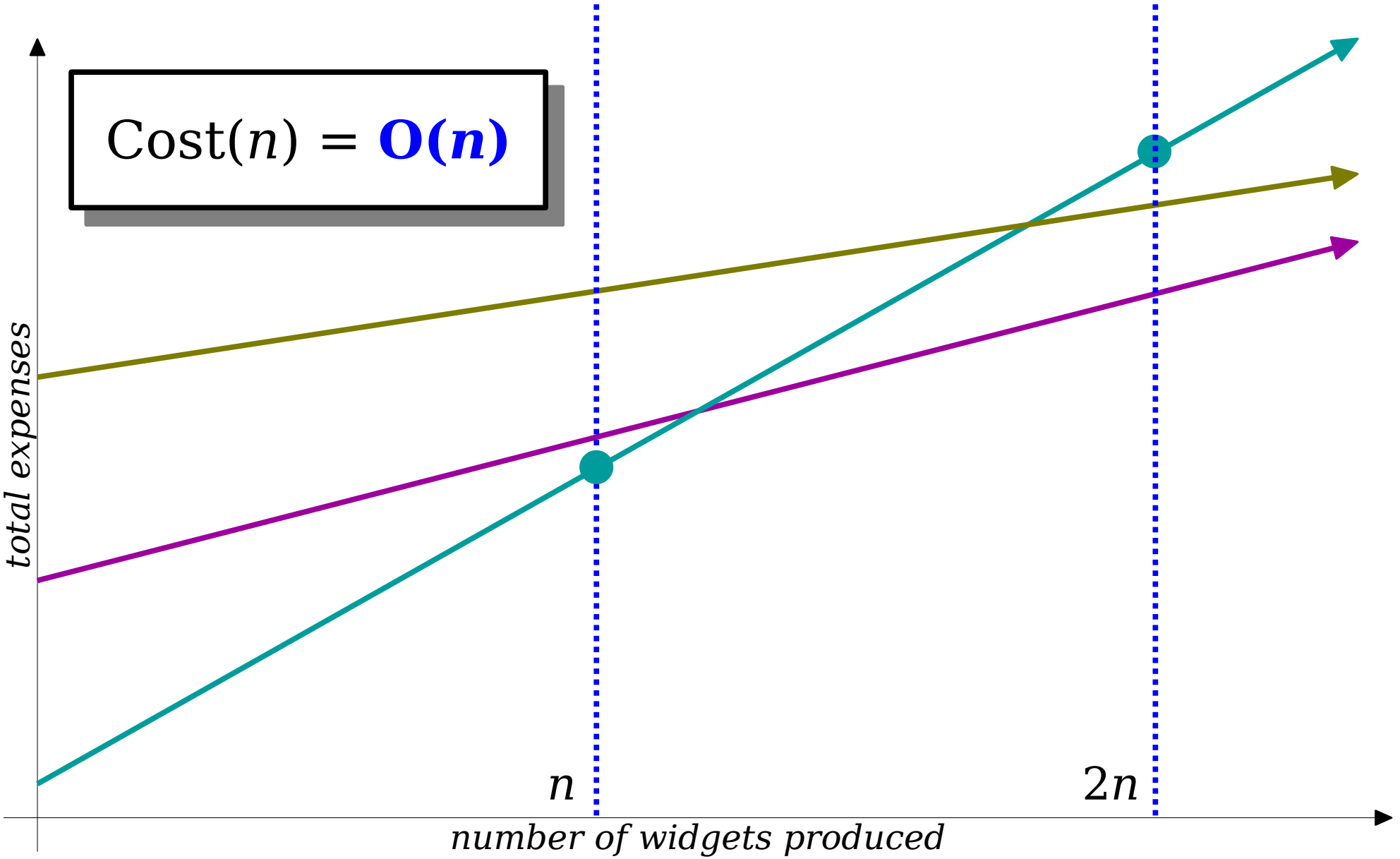

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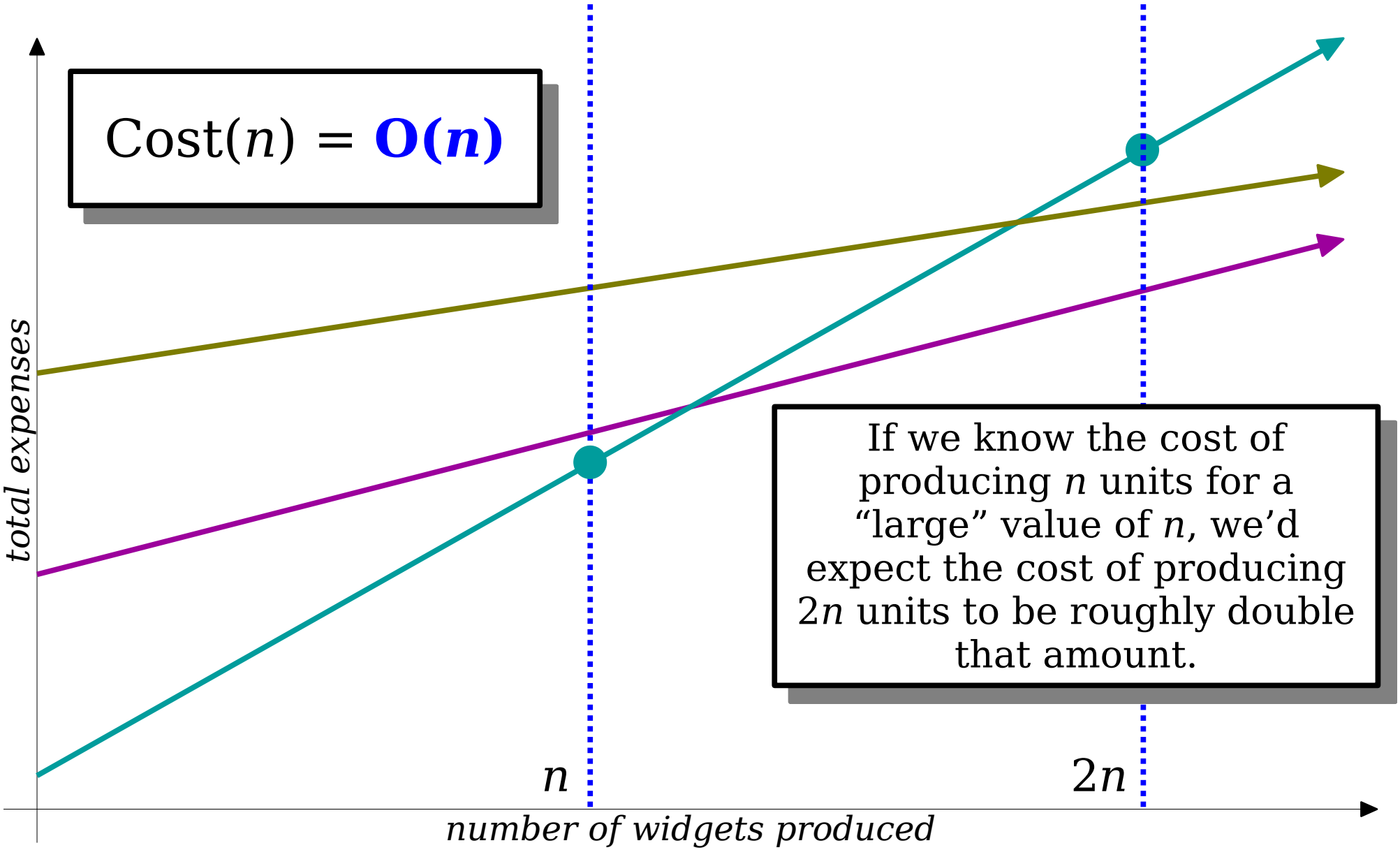

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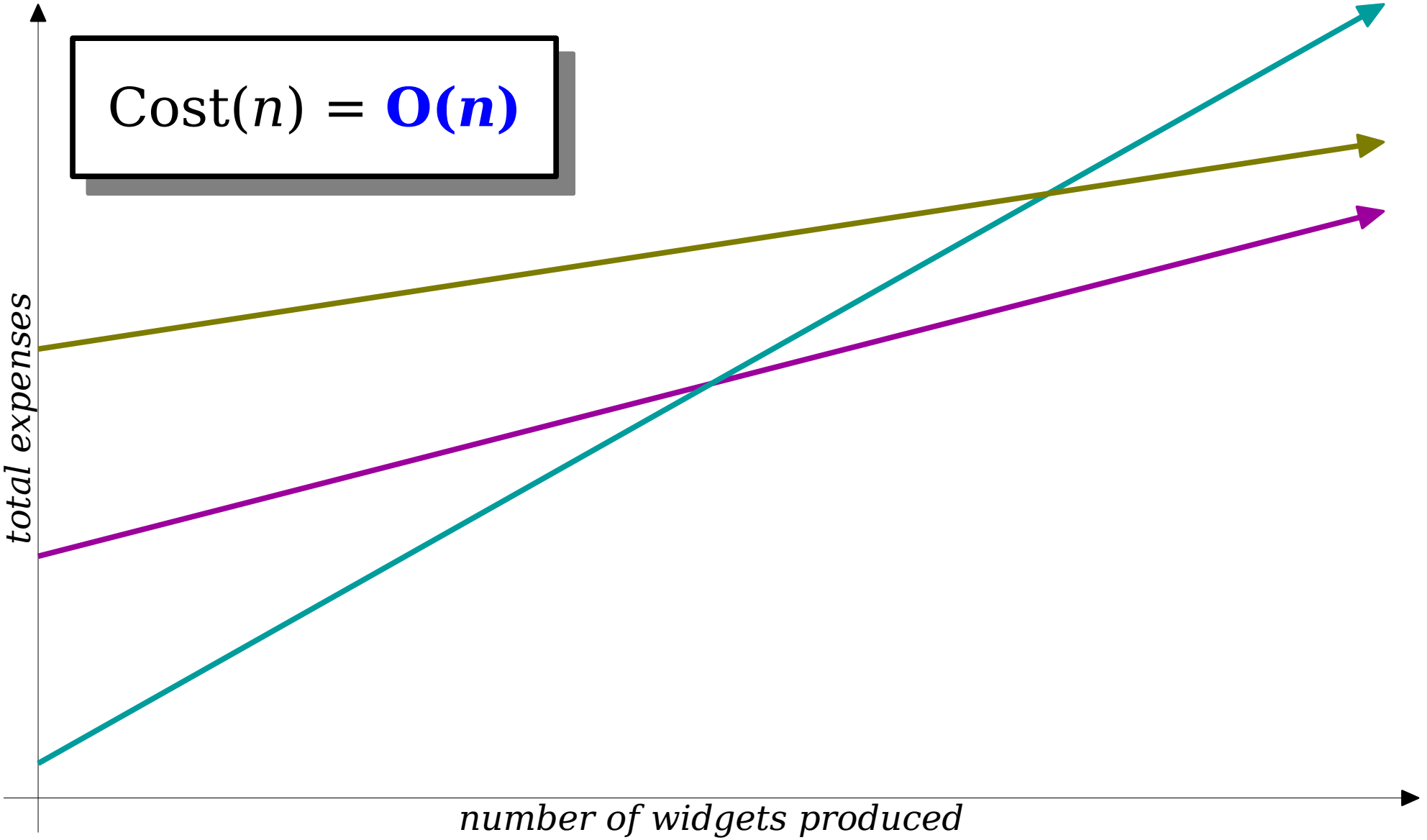

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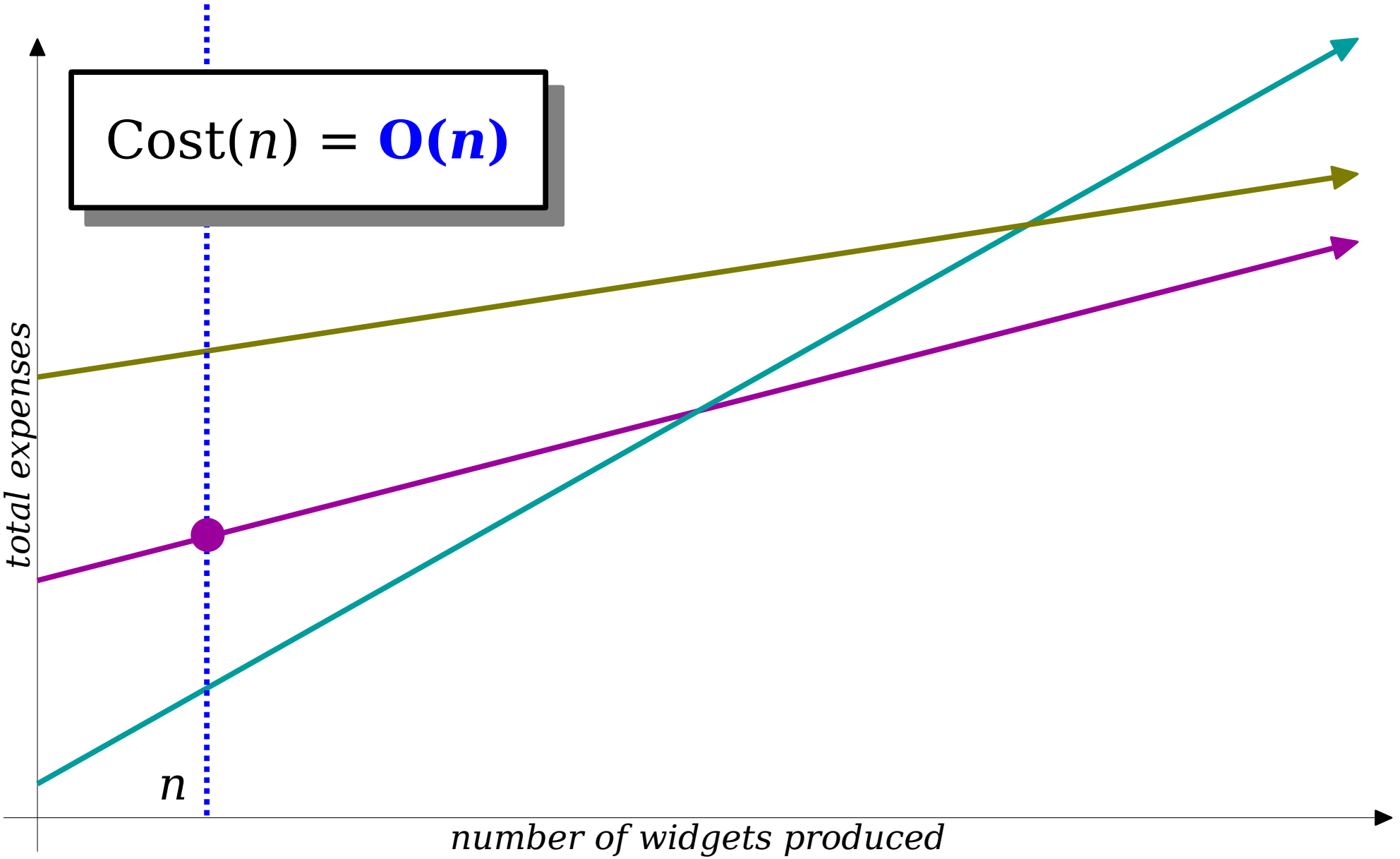

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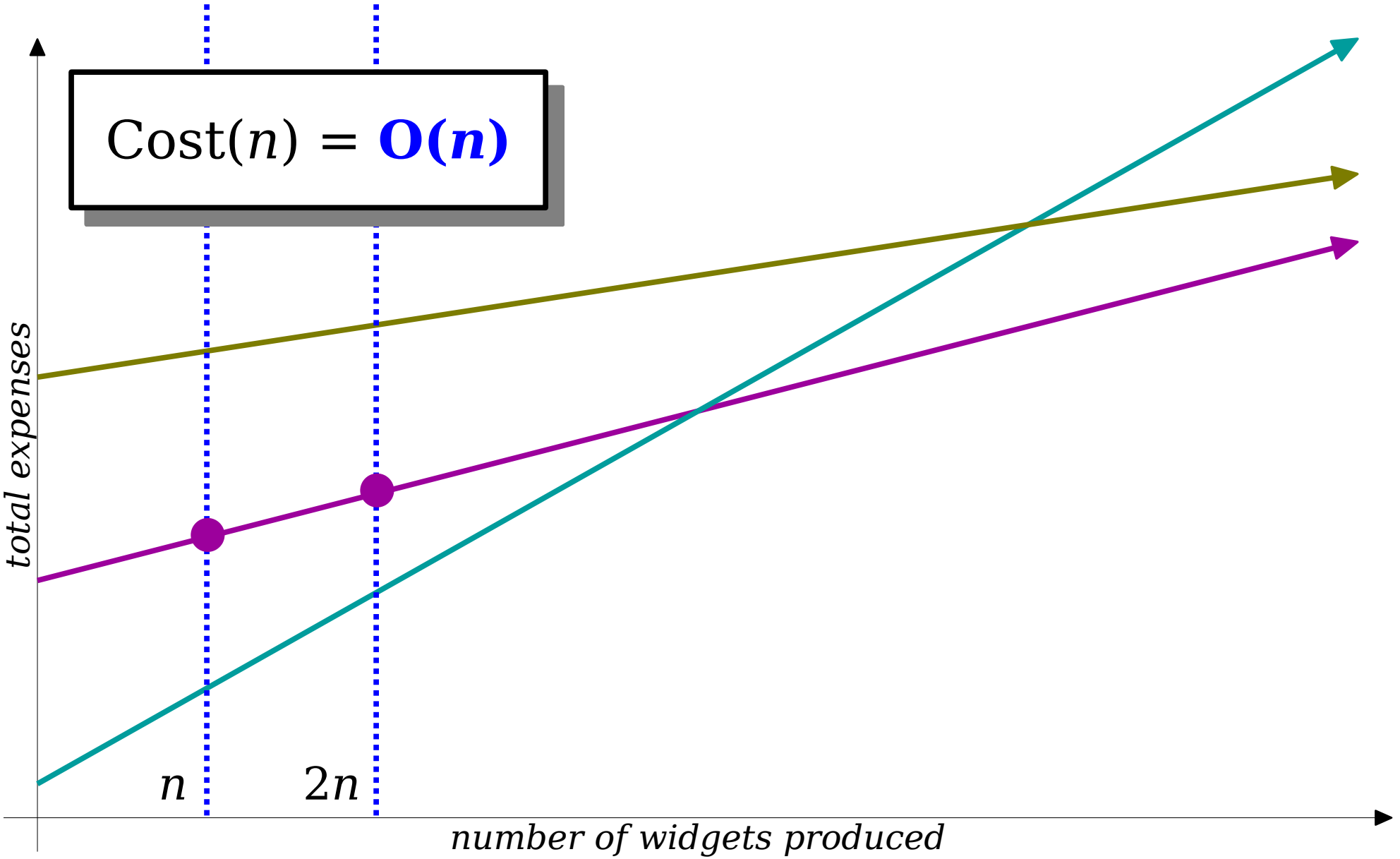

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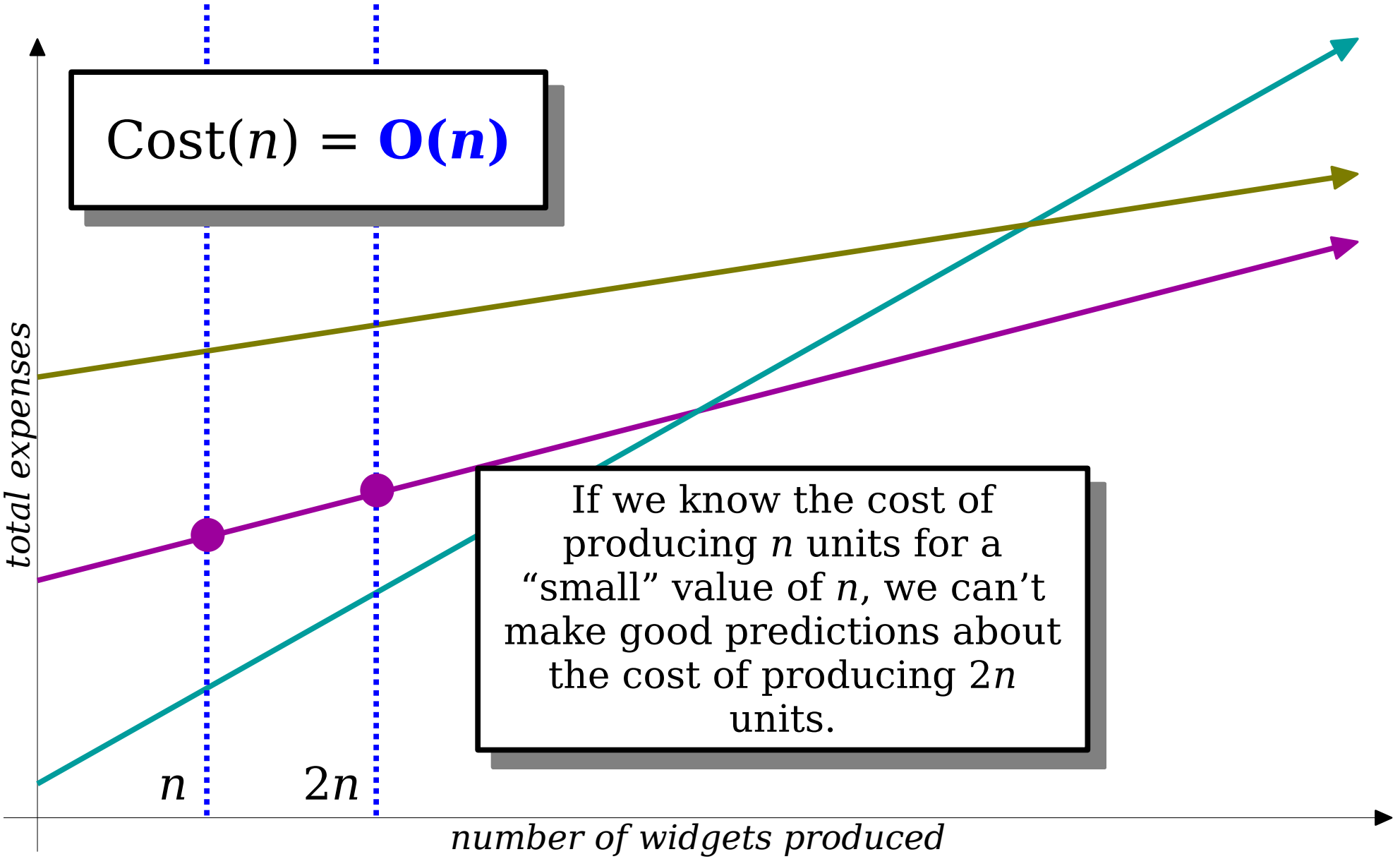

# Nuances of Big-O Notation

- Big-O notation is designed to capture the ***rate at which a quantity grows***.
- It does not capture information about
  - leading coefficients: the area of a square of side length $r$ and a circle of radius $r$ are each $O(r^2)$.
  - lower-order terms: the functions $n$, $5n$, and $137n + 42$ are all $O(n)$.
- However, it's still a powerful tool for predicting behavior.

# What does big-O notation have to do with computer science?

Time-Out for Announcements!

# Assignment 4

- Assignment 3 was due today at 1:00PM.
  - Need more time? You have four free "late days" to use over the quarter. You can use up to two of them here.
- Assignment 4 (***Recursion to the Rescue!***) goes out today. It's due next Friday at 1:00PM. You may work in pairs on this assignment.
  - Play around with recursive problem-solving in realistic situations.
  - Explore the power – and potential pitfalls – of recursive optimization.
- As always, feel free to ask for help when you need it! Ping us on EdStem, stop by the LaIR, visit our office hours, or email your section leader!

# Midterm Exam Reminder

- Our midterm exam will be on Monday, February 13th from 7:00PM – 10:00PM.
- We will go over more exam logistics this upcoming Monday. Briefly:
  - The exam covers L00 – L09 (basic C++ up through but not including recursive backtracking) and A0 – A3 (debugging through recursion).
  - It's a traditional sit-down, pencil-and-paper exam.
  - It's closed-book, closed-computer, and limited-note. You can bring an 8.5" × 11" sheet of notes with you to the exam.
- We've posted a huge searchable bank of practice problems to the course website, along with three practice exams made from questions selected from that bank.
- Students with OAE accommodations: If you need exam accommodations, please contact us ASAP if you haven't yet done so.

# *fg*

*(The Unix command to resume a program that was paused)*

# What does big-O notation have to do with computer science?

***Fundamental Question:***

How do we measure efficiency?

One Idea: ***Runtime***

# Why Runtime Isn’t Enough

- Measuring wall-clock runtime is less than ideal, since
  - it depends on what computer you’re using,
  - what else is running on that computer,
  - etc.
- Worse, ***individual runtimes can’t predict future runtimes***.

```
double averageOf(const Vector<int>& vec) {
  double total = 0.0;


  for (int i = 0; i < vec.size(); i++) {
      total += vec[i];
  }


  return total / vec.size();
}
```

---

Assume any individual statement takes one unit of time to execute. If the input `Vector` has $n$ elements, how many time units will this code take to run?

```
double averageOf(const Vector<int>& vec) {
1   double total = 0.0;

                1          n+1          n
    for (int i = 0; i < vec.size(); i++) {
        total += vec[i];
    }           n

    return total / vec.size();  1
}
```

Assume any individual statement takes one unit of time to execute. If the input `Vector` has $n$ elements, how many time units will this code take to run?

```
double averageOf(const Vector<int>& vec) {
    double total = 0.0;                    // 1

    for (int i = 0; i < vec.size(); i++) { // 1, n+1, n
        total += vec[i];                   // n
    }

    return total / vec.size();             // 1
}
```

Is this useful?

What does that tell us?

One possible answer: $3n + 4$.

```
double averageOf(const Vector<int>& vec) {
    double total = 0.0;

    for (int i = 0; i < vec.size(); i++) {
        total += vec[i];
    }

    return total / vec.size();
}
```

1 | 1 | $n+1$ | $n$ | $n$ | 1

Doubling the size of the input roughly doubles the runtime.

If we get some data points, we can extrapolate runtimes to good precision.

~~One possible answer: $3n + 4$.~~

More useful answer: **$O(n)$**.

```
void printStars(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            cout << '*' << endl;
        }
    }
}
```

---

How much time will it take for this code to run, as a function of $n$? Answer using big-O notation.

```
void printStars(int n) {
    for (int i = 0; i < n; i++) {
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```
void printStars(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            do a fixed amount of work;
        }
    }
}
```

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How much time will it take for this code to run, as a function of $n$? Answer using big-O notation.

```
void printStars(int n) {
    for (int i = 0; i < n; i++) {

        do O(n) units of work;

    }
}
```

---

How much time will it take for this code to run, as a function of $n$? Answer using big-O notation.

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void printStars(int n) {
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How much time will it take for this code to run, as a function of $n$? Answer using big-O notation.

```
void printStars(int n) {

    do O(n^2) units of work;

}
```

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How much time will it take for this code to run, as a function of $n$? Answer using big-O notation.

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void printStars(int n) {

    do O(n^2) units of work;

}
```

---

Answer: **$O(n^2)$**.

```
void printStars(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            cout << '*' << endl;
        }
    }
}
```

---

Answer: $\mathbf{O(n^2)}$.

Answer at
***https://pollev.com/cs106bwin23***

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            cout << '*' << endl;
        }
    }
}
```

If we time this code on input $n$, how much longer will it take to run on the input $2n$?

---

Answer: $\mathbf{O(n^2)}$.

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
            cout << '*' << endl;
        }
    }
}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

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        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

Answer at
***https://pollev.com/cs106bwin23***

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
            cout << '*' << endl;
        }
    }
}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

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        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
            do one unit of work;
        }
    }
}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
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    for (int i = 0; i < 8; i++) {
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---

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        }
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    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {

        do 5n units of work;

    }
}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

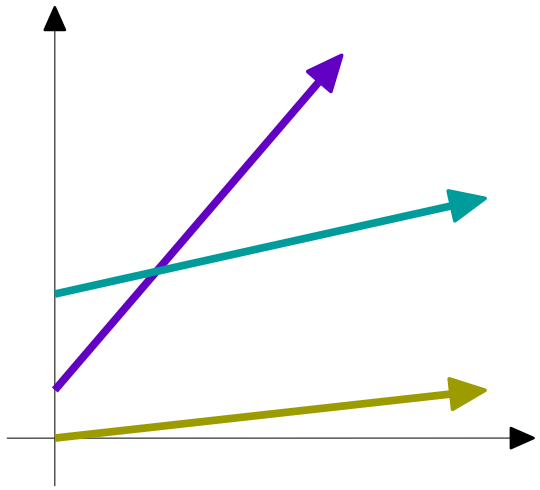
---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {

        do 5n units of work;

    }
}
```



Any linear function is O($n$). The leading coefficient is ignored.

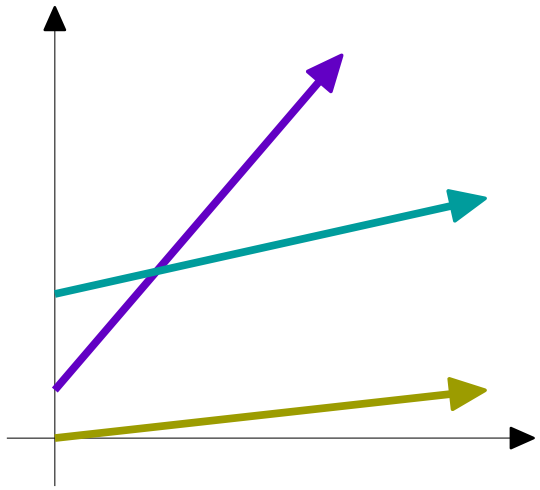
---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {

        do O(n) work;

    }
}
```



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How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {

        do O(n) work;

    }
}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {

    do 2n * O(n) work;


}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {

    do 2n * O(n) work;

}
```

As before, big-O ignores any leading coefficients.

```
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {

    do O(n²) work;

}
```

As before, big-O ignores any leading coefficients.

```
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
            cout << '*' << endl;
        }
    }
}
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

**$O(n^2)$**

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

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void beni(int n) {
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        }
    }
}
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    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
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}
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    for (int i = 0; i < 3 * n; i++) {
        do one unit of work;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
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            cout << '*' << endl;
        }
    }
}
void pando(int n) {

    do 3n units of work;

    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

$O(n^2)$

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
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        }
    }
}
void pando(int n) {

    do O(n) units of work;

    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
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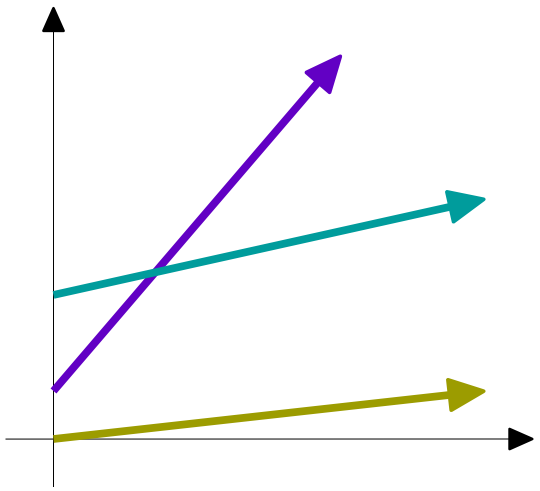

Any linear function is O($n$). The $y$-intercept doesn't matter.

---

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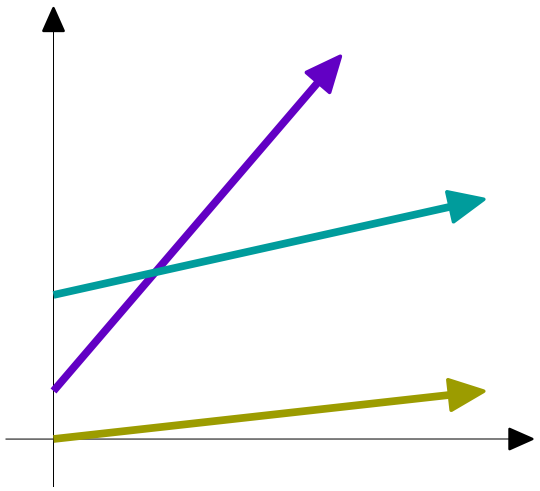

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$O(n^2)$

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```

**O($n^2$)**

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    }
}
```

**O($n$)**

---

How much time will it take for these functions to run, as a function of $n$? Answer using big-O notation.

# Recap from Today

- Big-O notation captures the rate at which a quantity grows or scales as the input size increases.
- Big-O notation ignores low-order terms and constant factors.
- “When in doubt, work inside out!” When you see loops, work from the inside out to determine the big-O complexity.

# Your Action Items

- ***Read Chapter 10.1 – 10.2.***
  - It's all about big-O and efficiency, and it's a great complement to what we covered today.
- ***Read the Guide to Big-O Notation.***
  - It includes a bunch of useful tips that expand upon what we did in lecture today.
- ***Start Assignment 4.***
  - If you want to follow our suggested timetable, aim to complete Win Sum, Lose Sum and Shift Scheduling by this Monday.

# Next Time

- ***Sorting Algorithms***
  - How do we get things in order?
- ***Designing Better Algorithms***
  - Using predictions from big-O notation.